SCEGGS Darlinghurst

2010

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Diagrams should be drawn in pencil
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Start each question in a new booklet

Total marks – 120

- Attempt Questions 1–10
- All questions are of equal value

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Attempt Questions 1–10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks)

(a) Evaluate *ln 27* correct to 2 decimal places. **2**

(b) Solve $|\, x+4 \,| = 3$. **2**

(c) Simplify $\dfrac{3}{x-1} - \dfrac{5}{x+1}$. **2**

(d) Solve: $\sqrt{3}\tan x = -1$ for $0 \le x \le 2\pi$. **2**

(e) Rationalise the denominator of $\dfrac{1}{\sqrt{3}-2}$. **2**

(f) Find a primitive function of $2+\dfrac{1}{x}$. **2**

End of Question 1

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Differentiate:

(i) $y = e^{2x}(3x - 2)$. **2**

(ii) $y = \log_e(2x^2 + 5)$. **2**

(b) Find $\int \sec^2 3x \; dx$. **1**

(c) Find the equation of the normal to the curve $y = x^2 - 4x$ at the point $(1, -3)$. **3**

(d) Evaluate $\sum_{r=1}^{3} (-1)^r (r+2)^2$. **2**

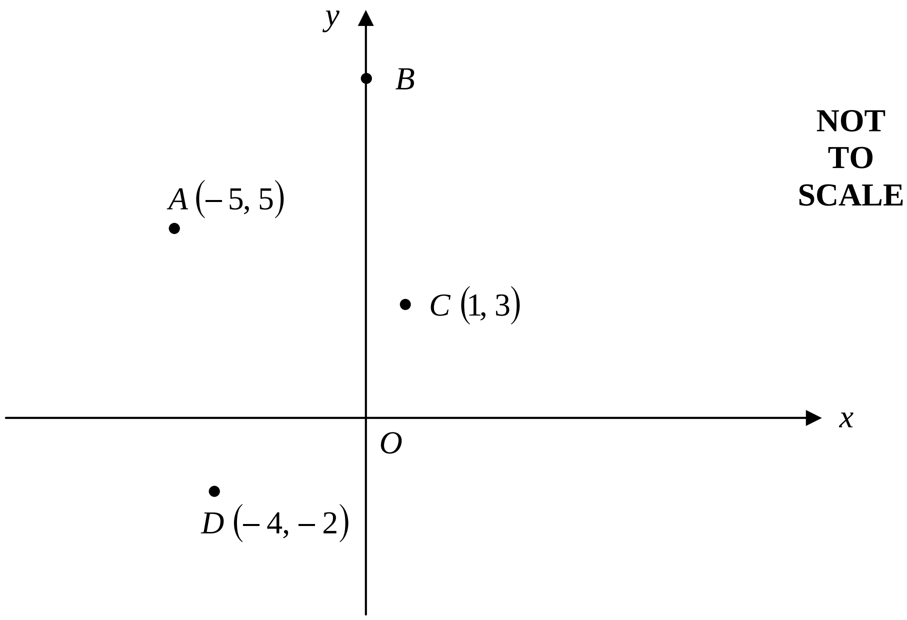
(e) Find the value of k if the sum of the roots of $x^2 - (k-1)x + 2k = 0$ is equal to the product of those roots. **2**

End of Question 2

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a)



The diagram shows the points $A(-5, 5)$ and $C(1, 3)$ and $D(-4, -2)$.
B is a point on the *y-axis.*

(i) Find the gradient of *AC*. **1**

(ii) Find the midpoint of *AC* **1**

(iii) Show that the equation of the perpendicular bisector of *AC* is $3x - y + 10 = 0$. **1**

(iv) Find the co-ordinates of *B* given that *B* lies on $3x - y + 10 = 0$. **1**

(v) Show that the point $D(-4, -2)$ lies on $3x - y + 10 = 0$. **1**

(vi) Show that *ABCD* is a rhombus. **2**

(b) (i) On the same set of axes sketch the graphs $y = 4 - x^2$ and $y = 3$. **2**

(ii) The graph $y = 3$ cuts the parabola at *A* and *B*. Find the co-ordinates of *A* and *B*. **1**

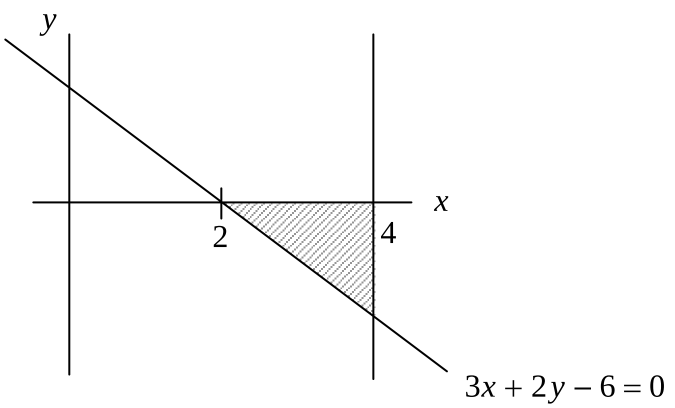
(iii) Calculate the area enclosed by the graphs $y = 4 - x^2$ and $y = 3$. **2**

End of Question 3

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) Write down three inequalities to describe the shaded region shown below. **2**



(b) Consider the function defined by $f(x) = x^3 - 6x^2 + 9x + 2$.

(i) Find $f'(x)$. **1**

(ii) Find the coordinates of the two stationary points. **2**

(iii) Determine the nature of the stationary points. **2**

(iv) Sketch the curve $y = f(x)$ for $0 \le x \le 4$ clearly labelling the stationary points. **2**

(v) Apply the Trapezoidal Rule with 5 function values to find an approximation to an area between $f(x) = x^3 - 6x^2 + 9x + 2$ and the x-axis between $x = 0$ and $x = 4$. **3**

End of Question 4

Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) By using the substitution of $y = 5^x$ solve for x the equation $25^x - 26(5^x) + 25 = 0$. **2**

(b) Sketch the curve that has the following properties. **3**

$f(2) = 1$
$f'(2) = 0$
$f''(2) = 0$
$f'(x) \geq 0$ for all real x.

(c) Solve $2\log_b x = \log_b 2 + \log_b (x+4)$. **3**

(d) (i) Show that $\frac{d}{dx}(x\log_e x) = \log_e x + 1$. **1**

(ii) Hence evaluate $\int_1^e (\log_e x)\ dx$. **3**

End of Question 5

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) Calculate the area of the region enclosed by the graph of $y = \cos 2x$, the x-axis and the ordinates at $x = 0$ and $x = \dfrac{\pi}{4}$. **2**

(b) Tom is an enthusiastic gardener. He planted a silky oak tree three years ago when it was 80 centimetres tall. At the end of the first year after planting, it was 130 centimetres tall, that is it grew 50 centimetres. Each year's growth was then 90% of the previous year's.

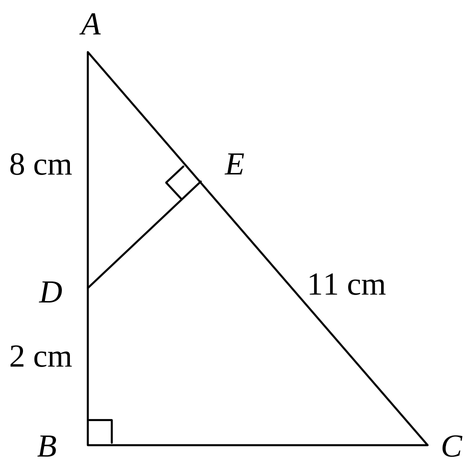
(i) What was the growth of the silky oak in the second year? **1**

(ii) How tall was the silky oak after three years? **1**

(iii) Assuming that it maintains the present growth pattern, explain why the tree will never reach a height of 6 metres. **2**

(iv) In which year will the silky oak reach a height of 5 metres? **2**

(c) *ABC* is a right-angled triangle in which $\angle ABC = 90°$. Points *D* and *E* lie on *AB* and *AC* respectively such that *AC* is perpendicular to *DE*. *AD* = 8 cm, *EC* = 11 cm and *DB* = 2 cm.



NOT TO SCALE

(i) Prove that ΔABC is similar to ΔAED. **2**

(ii) Find the length of *AE*. **2**

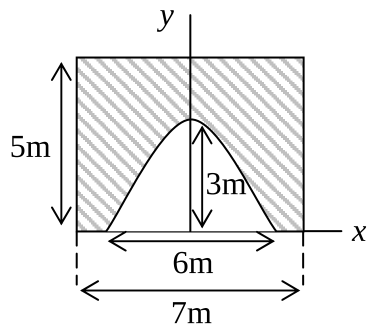
End of Question 6

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) Simplify $\dfrac{2\sec^2 A - 2}{2\tan A}$ **2**

(b) The diagram represents an archway of a building that is 5m high and 6m wide. The curved part is in the shape of a parabola with vertex 3m above the ground.



Use the axis shown in the diagram to:

(i) show that the equation of the parabola is $y = -\frac{1}{3}x^2 + 3$. **1**

(ii) find the shaded area. **3**

(c) The College gardener knows that the probability of a seedling growing to maturity is 0.95.

(i) If the gardener plants 2 seedlings, what is the probability that both will survive to maturity? **1**

(ii) If the gardener plants 5 seedlings, what is the probability that at least one seedling will die before reaching maturity? Express as decimal correct to 2 decimal places. **2**

(iii) If the gardener plants n seedlings, what is the maximum value of n if the probability that at least one seedling will die before reaching maturity is less than 0.5? **3**

End of Question 7

Marks

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a) For the function $y = 3\cos 4x - 5\sin x$, find the value of k if **3**

$$y + \frac{d^2y}{dx^2} = k\cos 4x.$$

(b) (i) Sketch the curve $y = \ln(x-1)$ clearly showing the x-intercept and the asymptote. **2**

(ii) The region enclosed by the curve $y = \ln(x-1)$ and the lines $x = 0$, $y = 0$ and $y = \ln 3$ is rotated about the y-axis to form a solid of revolution. Find the volume of this solid. **3**

(c) Jane and Ruby play a tennis match against each other. The probability in any set that Jane wins is $\frac{3}{5}$. The first player to win 2 sets wins the match.

(i) Find the probability that the match ends at the second set. **1**

(ii) Find the probability that Jane wins at least one set. **1**

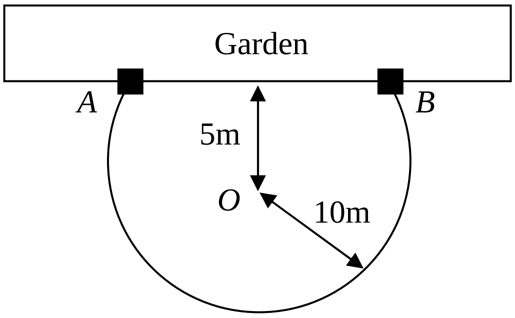
(iii) Find the probability that the person who wins the first set goes on to win the match. **2**

End of Question 8

Marks

Question 9 (12 marks) Use a SEPARATE writing booklet.

(a)



NOT TO SCALE

A water sprinkler covers a circular lawn area of radius 10 metres.
The sprinkler (*O*) is placed 5 metres from a rectangular garden bed.

(i) Garden stakes are placed at *A* and *B*. **1**

Show that $\angle AOB = \dfrac{2\pi}{3}$ radians.

(ii) Show that the total perimeter of the lawn area covered by the sprinkler is $\dfrac{40\pi}{3}$ m. **2**

(iii) Show that the area of the lawn that the sprinkler will cover is $\dfrac{200\pi}{3} + 25\sqrt{3}$ m^2. **3**

(b) Savannah buys a Porsche for \$320,000 and agrees to pay it off at the same amount each month over 8 years. The interest rate is 15% per annum, reducible monthly.

(i) If the monthly repayments are $\$M$, and $\$A_n$ is the amount owing after *n* repayments, show that the amount owing (in dollars) after the second repayment is given by: **1**

$$A_2 = 320\,000 \times 1.0125^2 - 1.0125\,M - M$$

(ii) Hence find the amount of each monthly repayment. **3**

Question 9 continues on page 10

Marks

Question 9 (continued)

(c) For what values of p does the equation $\sin x = px$ have two solutions in the domain $0 \leq x \leq \pi$. **2**

End of Question 9

Marks

Question 10 (12 marks) Use a SEPARATE writing booklet.

(a) Find the co-ordinates of the vertex of the parabola $y^2 - 6y - 9x - 9 = 0$. **2**

(b) John puts \$2 000 into a superannuation account on his 40th birthday.
He continues to do this on his birthday up to and including his 60th birthday.
The interest he earns is 10% pa compounded yearly.

On his 61st birthday he moves the accumulated amount into an account which earns 8% pa compounded yearly.

He will collect his accumulated amount on his 65th birthday.

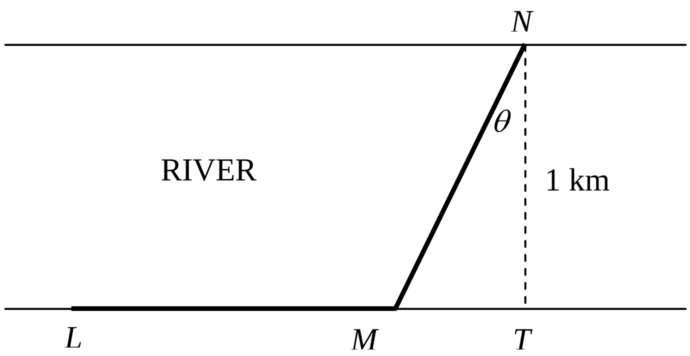
(i) How much does the first \$2 000 accumulate to when John celebrates his 61st birthday? **1**

(ii) How much will John collect on his 65th birthday? **3**

Question 10 continues on page 12

Question 10 (continued)

(c) It is desired to construct a cable link between two points L and N, which are situated on opposite banks of a river of width 1km. L lies 3km upstream from N. It costs 3 times as much to lay a length of cable underwater as it does to lay the same length overland. The following diagram is a sketch of the cables, where θ is the angle where NM makes with the direct route across the river.



NOT TO SCALE

(i) Prove $MN = \sec\theta$ and $MT = \tan\theta$. **1**

(ii) If segment LM costs c dollars per km, prove that the total cost (T) of laying the cable is given by **2**

$$T = 3c - c\tan\theta + 3c\sec\theta .$$

(iii) At what angle should the cable cross the river in order to minimize the total cost of laying it. **3**

End of Paper

Solutions to Trial 2010

Question 1

Calc 2

a) 3.2958 ✓
3.30 ✓

Well done

b) $x+4=3$
$x=-1$ ✓

$-(x+4)=3$
$-x-4=3$
$-x=7$
$x=-7$ ✓

c) $\frac{3}{x-1} - \frac{5}{x+1}$

$= \frac{3(x+1)-5(x-1)}{(x-1)(x+1)}$ ✓

$= \frac{3x+3-5x+5}{(x-1)(x+1)}$

$= \frac{x+8}{(x-1)(x+1)}$ ✓

Be careful of the bracket $-5(x-1) = -5x+5$

d) $\tan x = -\frac{1}{\sqrt{3}}$ Tan −ve Q 2+4

acute ∡ $= \frac{\pi}{6}$

$x = \frac{5\pi}{6}$ ✓, $\frac{11\pi}{6}$ ✓

e) $\frac{1}{\sqrt{3}-2} \cdot \frac{\sqrt{3}+2}{\sqrt{3}+2}$

$= \frac{\sqrt{3}+2}{3-4} = -\sqrt{3}-2$

alot of careless errors made here.
Denominator $= -1$
$\therefore \frac{\sqrt{3}+2}{-1} = -1(\sqrt{3}+2) = -\sqrt{3}-2$

Not $-\sqrt{3}+2$

f) $y' = 2 + \frac{1}{x}$ Calc 2

$y = 2x + \ln x + c$

$\int \frac{1}{x} = \ln x$

1mk awarded for $2x$ $+c$

1mk awarded for $\ln x$.

Cal /5   Reas /7

Question 2

(a) (i) $y = e^{2x}(3x-2)$ ✓
$y' = e^{2x}(3) + (3x-2)e^{2x}.2$ ✓   Cal 2
$= e^{2x}[3+6x-4]$
$= e^{2x}[6x-1]$ (ise)

Those that identified Q as product rule handled it well. Ignored subsequent errors in factorisation + simplifying

(ii) $y = \log_e(2x^2+5)$
$y' = \frac{4x}{2x^2+5}$ ✓ ✓   Cal 2

Mostly well done. Most students got 4x for 1/2 mks.

(b) $\int \sec^2 3x\,dx = \frac{\tan 3x}{3} + c$ ✓   Cal 1

No penalty for not having "+c"

(c) $y = x^2 - 4x$
$y' = 2x-4$
$\therefore$ slope of tangent at $x=1$ Reas 3
$y' = 2(1)-4 = -2$
$\therefore$ slope of normal at $x=1$
$m = \frac{1}{2}$ ✓ Point $(1,-3)$

$y+3 = \frac{1}{2}(x-1)$ ✓

$2y+6 = x-1$
$x-2y-7=0$ ✓

mostly well done

Some students didn't realise it was the normal.

(d) $\sum_{r=1}^{3} (-1)^r (r+2)^2$ Reas 2

$= (-1)^1(3)^2 + (-1)^2(4)^2 + (-1)^3(5)^2$ ✓
$= -9+16-25$
$= -18$ ✓

Students need to be able to identify Σ as sum.

This is not an AP so sum formula can't be used

(e) $x^2-(k-1)x+2k=0$

$a=1$

$b=-(k-1)$

$c=2k$

Sum $=\frac{-b}{a}$ Product $=\frac{c}{a}$

$\frac{k-1}{1}=\frac{2k}{1}$ ✓

$\therefore -1 = k$ ✓

Reas 2

careless errors made with sum $=-\frac{b}{a}$ be careful of the signs.

Question 3

Cal 2
Com 2
Reas 2

a) i) $\frac{3-5}{1+5}=\frac{-2}{6}=-\frac{1}{3}$ ✓

ii) $\left(\frac{-5+1}{2}, \frac{5+3}{2}\right)$

$=(-2,4)$ ✓

iii) $m=3$ pt$=(-2,4)$

$y-y_1=m(x-x_1)$

$y-4=3(x--2)$

$y-4=3x+6$

$3x-y+10=0$ ✓

Do not assume that all references to the word "perpendicular" imply the use of the perpendicular distance formula!

iv) $(0,y)$ let $x=0$

$-y=-10$

$y=10$

$\therefore B(0,10)$ ✓

v) $3(-4)-(-2)+10=0$

$0=0$

$\therefore D(-4,-2)$ lies on the line $3x-y+10=0$ ✓

vi)

Midpoint BD

$\left(\frac{-4}{2}, \frac{10-2}{2}\right)$

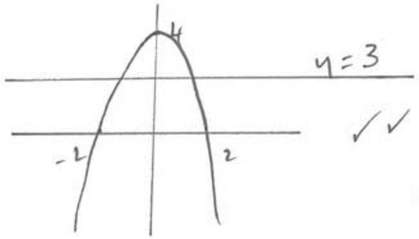
$=(-2,4)$ = Midpoint AC ✓

$\therefore$ ABCD is a rhombus as diagonals bisect each other at $(-2,4)$ ✓

Reas 2

Many students went on "fishing" expeditions – listing all the properties of a rhombus that they could think of... Full marks required a clear, precise response.

b) i)



✓✓

Com 2

ii)

$3 = 4 - x^2$

$x^2 = 1$

$x = \pm 1$

When $x = 1$ $y = 3$

$x = -1$ $y = 3$

A $(-1, 3)$ B $(1, 3)$ ✓

iii)

$A = \int_{-1}^{1} (4 - x^2)\,dx - 3\,dx$ ✓

$= \int_{-1}^{1} 1 - x^2 \, dx$

$= \left[x - \frac{x^3}{3} \right]_{-1}^{1}$

Calc 2

$= \frac{4}{3}$ units^2 ✓

Well done.

Calc 4
Com 2
Reas 2

Question 4

(a)

$x \le 4$
$y \le 0$ } ✓

$3x + 2y - 6 \ge 0$ ✓

Reas 2

Poorly done. Few students realised that the unbroken boundaries required both equality and inequality signs. Few students were able to test the points in an appropriate manner.

(b) $f(x) = x^3 - 6x^2 + 9x + 2$

(i) $f'(x) = 3x^2 - 12x + 9$ ✓

(ii) For stat point $f'(x) = 0$

$3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1, 3$

Calc 2

Need both x and y values!

$(1, 6)$ ✓ and $(3, 2)$ ✓

(iii) For nature $f''(x)$

Calc 2

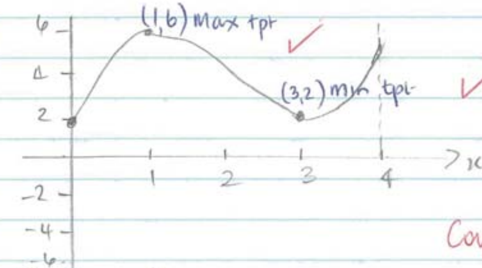
$f''(x) = 6x - 12$

At $x = 1$ $f''(1) = -6$ $\therefore (1, 6)$ max ✓

At $x = 3$ $f''(3) = +6$ $\therefore (3, 2)$ Min ✓

Tables of — / ‾ \ — are worthless without a clear supporting statement of the nature of the points.

(iv)



✓ ✓

Many students did not deal adequately with the end points.

Com 2

$f(4) = 6$

(v) See over

x	0	1	2	3	4
$f(x)$	2	6	4	2	6
Fact	1	2	2	2	1
Prod	2	12	8	4	6

✓

$\Sigma = 32$

$A \doteqdot \frac{h}{2} \times \Sigma$ ✓

$= \frac{1}{2} \times 32$

$= 16 \text{ units}^2$ ✓

Still some students with incorrect value for 'h'. h = width of strip

or

$h = \frac{b-a}{n}$

$n = 4, n \neq 5.$

Some students could not apply the correct factors.

Question 5

Calc 4
Com 3
Reas 5

a) $y = 5^x$

$5^{2x} - 26(5^x) + 25 = 0$

$y^2 - 26y + 25 = 0$

$(y - 25)(y - 1) = 0$ Reas 2

$y = 25 \quad y = 1$

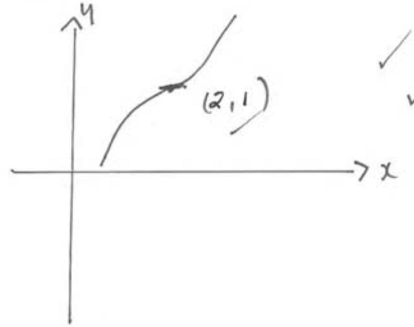
$5^x = 25 \qquad 5^x = 1$

$5^x = 5^2$ ✓

$x = 2$ or $x = 0$ ✓

Mostly well done.

b)



✓ ✓ Com 3

1 for positive gradient
1 for correct point
1 for horizontal inflexion
(even if drawn correctly) inflexion point (without horizontal) was not awarded a mark.

c) $\log_b x^2 = \log_b 2(x+4)$ ✓

$x^2 = 2(x+4)$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$ ✓ Reas 3

$x = 4 \qquad x = -2$ No soln

$\therefore x = 4$ ✓

Very poor log laws.

Note that $x > 0$ as $\log_b x$ is defined for $x > 0$ only.

d)

i) $\frac{d}{dx}(x \log_e x)$

$\frac{dy}{dx} = x \cdot \frac{1}{x} + \log_e x \cdot 1$ Calc 4

$= \log_e x + 1$ ✓

ii) $\int \log_e x \, dx = x \log_e x - \int 1 \, dx$ ✓

$= \left[x \log_e x - x\right]_1^e$ ✓

$= \left[e \log_e e - e\right] - \left[\log_e 1 - 1\right]$

$= e \cdot 1 - e - 0 + 1$

$= 1$ ✓

Well done but make working absolutely clear.

not well done. Must show that the result of part (i) has been used. $\int_1^e 1 \, dx$ was often forgotten.

Calc 2
Reas 10

Question 6

(a) $y = \cos 2x$

$A = \int_0^{\frac{\pi}{4}} \cos 2x \, dx$

$= \left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}}$ ✓ Calc 2

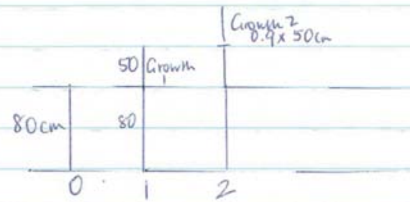
$= \frac{1}{2}\left(\sin \frac{\pi}{2} - \sin 0\right)$

$= \frac{1}{2}(1)$

$= \frac{1}{2}$ unit^2 ✓

Well done.

(b)



Reas 6

(i) Growth in second years

$= 0.9 \times 50$

$= 45$ cm ✓

(ii) Height after 3 years

= initial height + Growth 1 + Growth 2 + Growth 3

$n = 3 \quad a = 50 \quad r = 0.9$ $\quad S_n = \frac{a(1-r^n)}{1-r}$

$= 80 + \frac{50(1-0.9^3)}{1-0.9}$

$= 215.5$ cm ✓

It was important to note that 80 cm was not the first term of the sequence. The question refers to growth.

(iii) the sequence of growths is a GP with $r = 0.9$. there will be a limiting sum

$a = 50 \quad r = 0.9$

$S_\infty = \frac{50}{1-0.9} = 500 \text{ cm} = 5\text{m}$ ✓

∴ the maximum growth is 5m

the maximum height = 5.8 m

∴ the tree will never reach 6m. ✓

must show calculations.

must add the 80cm to the infinite sum.

(iv) If the tree reaches 5 m, its growth would be 4.2 m

$\therefore S_n = 420 \qquad a = 50$

$420 = 50\left(\frac{1-0.9^n}{0.1}\right) \qquad r = 0.9$

$\frac{420 \times 0.1}{50} = 1 - 0.9^n$ ✓

$0.9^n = 1 - \frac{420 \times 0.1}{50}$

$= 4/25$

$n = \frac{\ln(4/25)}{\ln(0.9)}$

$= 17.3$ ✓

∴ tree will reach height of 5m in the 18th year.

$a = 50$

$r = 0.9$

The sum is 420cm not 500cm

no marks deducted for 17th year

(c) (i) In ΔABC, ΔAED

$\angle A$ is common

$\angle AED = \angle ABC$ Given

$= 90$ ✓

Reas 2

∴ $\Delta ABC ||| \Delta AED$ (AA test or equiangular) ✓

$\Delta \widehat{ABC} ||| \Delta \widehat{AED}$

(ii) $\frac{AE}{AD} = \frac{AB}{AC}$

$\frac{AE}{8} = \frac{10}{AE+11}$ ✓

$AE^2 + 11AE = 80$

$AE^2 + 11AE - 80 = 0$

$(AE+16)(AE-5) = 0$

$AE = -16$ (impossible)

$AE = 5$ ✓

Reas 2.

Well done Reason required.

Better to draw the triangles separately. to find corresponding sides.

Question 7

Cal 3
Reas 9

a) $\dfrac{2(\sec^2 A - 1)}{2\tan A}$

Many students made this much more complicated than it needed to be.

$\dfrac{\tan^2 A + 1 - 1}{\tan A}$ ✓ Reas 2 ✓

$= \tan A$

b) Test $(0,3)$ $(3,0)$ $(-3,0)$

$y = -\frac{1}{3}x^2 + 3$ $0 = -\frac{1}{3}(3)^2 + 3$ $0 = -\frac{1}{3}(-3)^2 + 3$

$3 = 3$ $0 = 0$ $0 = 0$ ✓ Reas 1

Since all 3 points are on the parabola the equation is $y = -\frac{1}{3}x^2 + 3$

poorly done! the easiest way to show this is to substitute 3 points and test to see if they lie on the curve.

ii) $A = 5 \times 7 - 2\int_0^3 \left(-\frac{1}{3}x^2 + 3\right) dx$ ✓

$= 35 - 2\left[-\frac{x^3}{9} + 3x\right]_0^3$ ✓

$= 35 - 2\left(-\frac{27}{9} + 9 - 0\right)$ Calc 3

$= 23 \text{ units}^2$ ✓

5x6 was also accepted for the rectangle as these were the measurements given in the wording of the question

c) i) P(both survive) $= 0.95 \times 0.95$
$= 0.9025$

ii) $1 - (0.95)^5$ Reas 6.

$= 0.23$

iii) $1 - (0.95)^n < 0.5$
$-(0.95)^n < -0.5$
$(0.95)^n > 0.5$
$n \log 0.95 > \log 0.5$
$n < \dfrac{\log 0.5}{0.95}$ $n < 13.5$

This question was fairly well done. part (iii) there was some confusion with the inequality sign. Note: when $0 < a < 1$ in $\log a$ then $\log a$ is negative hence dividing by $\log 0.95$ the sign must be reversed.

Calc 6
Com 2
Reas 4

Question 8

(a) $y = 3\cos 4x - 5\sin x$
$y + \dfrac{d^2y}{dx^2} = k\cos 4x$ Calc 3

$\dfrac{d^2y}{dx^2}$ means the second derivative and many students didn't know this.

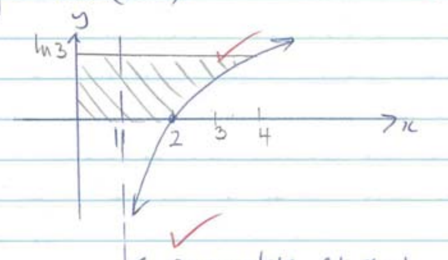
$\dfrac{dy}{dx} = -3 \times 4\sin 4x - 5\cos x$ ✓

$\dfrac{d^2y}{dx^2} = -48\cos 4x + 5\sin x$ ✓

$\therefore 3\cos 4x - 5\sin x - 48\cos 4x + 5\sin x = k\cos 4x$

$-45\cos 4x = k\cos 4x$
$\therefore k = -45$ ✓

(b) (i) $y = \ln(x-1)$ Com 2



← asymptote at $x = 1$

(ii) $V = \pi\int x^2\, dy$ $y = \ln(x-1)$
$x - 1 = e^y$
$V = \pi\int_0^{\ln 3} e^{2y} + 2e^y + 1\, dy$ $x = e^y + 1$ ✓
$x^2 = (e^y + 1)^2$

$= \pi\left[\dfrac{e^{2y}}{2} + 2e^y + y\right]_0^{\ln 3}$ ✓

$= \pi\left[\dfrac{e^{2\ln 3}}{2} + 2e^{\ln 3} + \ln 3 - \dfrac{e^0}{2} - 2e^0\right]$

Volume around the y axis is given by $V = \pi\int x^2\, dy$ $\therefore$ you must rearrange and find an expression for x^2. Expand this before integrating.

$$= \pi \left[\frac{e^{2\ln 3}}{2} + 2e^{\ln 3} + \ln 3 - \frac{1}{2} - 2 \right]$$

$$= \pi \left[\frac{e^{2\ln 3}}{2} + 2e^{\ln 3} + \ln 3 - \frac{5}{2} \right]$$

Calc 3

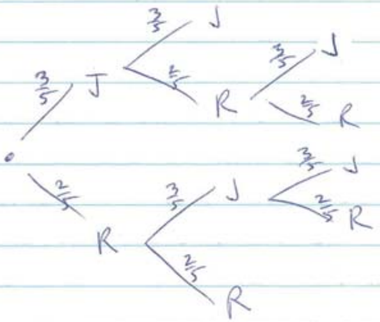
$$= \pi \left[\frac{e^{\ln 9}}{2} + 2e^{\ln 3} + \ln 3 - \frac{5}{2} \right]$$

$$= \pi \left[\frac{9}{2} + 2\times(3) + \ln 3 - \frac{5}{2} \right]$$

$$= \pi \left[\frac{9}{2} + 6 + \ln 3 - \frac{5}{2} \right]$$

$$= \pi \left[8 + \ln 3 \right] \quad \text{or} \quad 28.6 \text{ units}^3$$ ✓

(c)



Students who answered this successfully were those that drew a tree diagram or wrote down the required selections for each question.

(i) P(JJ) or P(RR)

$\left(\frac{3}{5}\right)^2 + \left(\frac{2}{5}\right)^2 = \frac{13}{25}$ ✓

Reas 4

(ii) 1 − P(RR)

$1 - \left(\frac{2}{5}\right)^2 = \frac{21}{25}$ ✓

(iii) P(JJ) or P(JRJ) or P(RJR) or P(RR)

$\left(\frac{3}{5}\right)^2 + \frac{3}{5}\times\frac{2}{5}\times\frac{3}{5} + \frac{2}{5}\times\frac{3}{5}\times\frac{2}{5} + \left(\frac{2}{5}\right)^2 = \frac{19}{25}$ ✓ ✓

Question 9

Reas 9.

a)

i) $\cos\theta = \frac{5}{10} = \frac{1}{2}$ $\quad 2\times\theta = \frac{2\pi}{3}$ ✓

$\theta = \frac{\pi}{3}$

ii) $2\times\pi\times 10 - \left(10\times\frac{2\pi}{3}\right)$ ✓

$20\pi - \frac{20\pi}{3}$

$\frac{60\pi - 20\pi}{3}$

$= \frac{40\pi}{3}$ ✓

alternative soln was $2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$

Then using arc length

$l = 10\times\frac{4\pi}{3}$

$= \frac{40\pi}{3}$

iii) Area Circle − Area minor segment

$\pi r^2 - \left[\frac{1}{2}r^2(\theta - \sin\theta)\right]$

$\pi 100 - \left[50\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)\right]$ ✓

$100\pi - \frac{100\pi}{3} + 50\sin\frac{2\pi}{3}$

$\frac{300\pi - 100\pi}{3} + 50\sin\frac{2\pi}{3}$ ✓

$\frac{200\pi}{3} + 50\times\frac{\sqrt{3}}{2}$

Reas 3

$= \frac{200\pi}{3} + 25\sqrt{3}$ ✓

Various solutions accepted

Area of sector + Δ but working required to get mark for base of triangle if used $\frac{1}{2}b\times h$.

fudging to get solution ⇒ no marks.

b) Reas 4

$n = 8 \times 12 = 96$

$r = 0.15 \div 12 = 0.0125$

$A_1 = 320{,}000(1.0125) - M$

$A_2 = [320{,}000(1.0125) - M](1.0125) - M$ ✓

$= 320{,}000(1.0125)^2 - 1.0125M - M$

$= 320{,}000(1.0125)^2 - M[1.0125 + 1]$

$A_{96} = 320{,}000(1.0125)^{96} - M[1.0125^{95} + \dots 1]$ ✓

$a = 1$, $r = 1.0125$, $n = 96$

$A_{96} = 0$

$320000(1.0125)^{96} = M\left[\frac{1(1.025^{96} - 1)}{0.125}\right]$ ✓

$\frac{320000(1.0125)^{96}(0.025)}{1.0125^{96} - 1} = M$

$\$5742.53 = M$ ✓

c) if $p \geq 1$, then the line won't intersect twice

$p \leq 0$, when gradient $= 0$, the line $y = 0$ cuts the graph twice!

for $0 \leq p < 1$ Reas 2

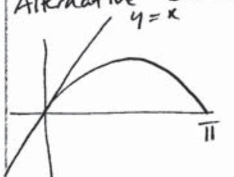
the equation $\sin x = px$ has two solutions.

Students needed to show connection with A_1 either as shown or $A_2 = A_1(1.0125 - M)$ to be awarded marks

Most students handled this well.

Careless errors with gp and $n \neq 95$.

Alternative solutions



$y = \sin x$

$y' = \cos x$

An attempt at showing this graphically, students were awarded 1 mk.

Calc 3

Reas 7

Question 10

(a) $y^2 - 6y - 9x - 9 = 0$

$y^2 - 6y + (3)^2 = 9x + 9 + 9$ ✓

$(y-3)^2 = 9(x+2)$

Vertex $= (-2, 3)$ ✓

Most (but not all) students could successfully complete the square.

(b) 21 deposits of \$2000

$r = 0.1$

(i) the first \$2000 matures in 21 years

$A_1 = P(1+r)^n$

$= 2000(1.1)^{21}$

$= \$14\,800.50$ ✓

Only a few students recognized that there were 21 deposits.

Several students did not see the annuity context was followed by a compound interest context.

(ii) Reas 4

$A_1 = 2000(1.1)^{21}$

$A_2 = 2000(1.1)^{20}$

$A_3 = 2000(1.1)^{19}$

$\vdots$

$A_{21} = 2000(1.1)^0$

$A_{TOT} = 2000(1.1)^{21} + 2000(1.1)^{20} + \dots + \dots 2000(1.1)^0$

$= 2000[1.1^{21} + 1.1^{20} + \dots 1]$ ✓

$= 2000\left[\frac{1(1.1^{22} - 1)}{0.1}\right]$

$= \$142{,}805.49$ ✓

This amount is then exposed to compound interest @ 8% p.a for 4 yrs

$A = 142805.99(1.08)^4$ ✓

$= \$194\,285.30$

Many students who had attempted rote learning of a procedure could not apply to given question.

Several students thought that 10% = 0.01 !!

(c) (i) $\sec\theta = \frac{MN}{1}$ $\tan\theta = \frac{MT}{1}$ Reas 1 ✓

$\therefore MN = \sec\theta$ $\therefore MT = \tan\theta$

Well done – many students we able to access these 3 marks.

(ii) $LM = LT - MT$

$= 3 - \tan\theta$ Reas 2 ✓

cost of $LM = (3-\tan\theta)c$

$MN = \sec\theta$

Cost of $MN = 3c \times \sec\theta$

$= 3c\sec\theta$ ✓

$\therefore$ total cost $= (3-\tan\theta)c + 3c\sec\theta$

$= 3c - c\tan\theta + 3c\sec\theta$

(iii) Minimise cost

$T = 3c - c\tan\theta + 3c\sec\theta$

$T = 3c - c\tan\theta + 3(\cos\theta)^{-1}c$

$T' = -c\sec^2\theta + 3c(\cos\theta)^{-2} \times \sin\theta$

$= -c\sec^2\theta + \frac{3c\sin\theta}{\cos^2\theta}$ Calc 3

$= c\left(\frac{-1}{\cos^2\theta} + \frac{3\sin\theta}{\cos^2\theta}\right)$

$= c\left(\frac{3\sin\theta - 1}{\cos^2\theta}\right)$ ✓

Only a few students could correctly differentiate T. Fewer still could solve $T'=0$. Many students calculated $\frac{dT}{dc}$ instead of $\frac{dT}{d\theta}$.

For max/min $T' = 0$

ie $3\sin\theta - 1 = 0$

$\sin\theta = \frac{1}{3}$ ✓

$\theta = \sin^{-1}\left(\frac{1}{3}\right) = 0.3398^c$

Check minimum value

θ	0.3^c	0.339^c	0.4^c
T'	-0.12	0	0.2
	\	—	/

$\therefore$ Minimum ✓

Several students did not demonstrate this adequately.